HISTORICAL NOTE.

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[Contributed by the request of the authors.]

If the lengths of the sides of a triangle are, respectively, 3, 4, 5 units, then the figure is a right triangle. This fact was known to the early Egyptians, who, it appears, based upon it a method of laying out their temples. They determined a N. and S. line by accurate astronomical observation, then ran a line at right angles to this by means of a rope stretched around three pegs in such a way that the three sides of a triangle thus formed were to each other as 3 : 4 : 5, one of the legs of the right triangle being made to coincide with the N. and S. line.* Essentially the same process was described later by Heron of Alexandria, by the Hindu astronomers, and by Chinese writers. The Hindus took for the lengths of the sides 15, 36, 39, respectively. There is reason to believe that the Egyptian "rope-stretchers" existed as early as the time of King Amenemhat I., about 2300 B. C. If this date is correct, then this method of laying out right angles in the field by rope-stretching was in vogue fully 3000 years !

The discovery of the well-known property of the right triangle is ascribed by Greek writers to Pythagoras. The truth of the theorem for the special case when the sides are 3, 4, 5, respectively, he may have learned from the Egyptians. That the importance and beauty of this theorem of three squares was thoroughly appreciated by the Greeks is evident from the legend to which its discovery gave rise. Pythagoras is said to have been so jubilant over his great achievement, that he offered a hecatomb to the muses who inspired him. As the Pythagoreans believed in the transmigration of the soul and, for that reason, opposed the shedding of blood, the sacrifice was replaced in the traditions of the Neo-Pythagoreans by that of "an ox made of flour"! The proof given by Pythagoras for this theorem has not been handed down to us. That in *Euclid* I, 47 is due to Euclid himself. Much ingenuity has been expended in conjecture as to the nature of the proof given by Pythagoras. Some critics believe that the proof involved the consideration of special cases ; that it was essentially that for the isosceles right triangle outlined by Plato in *Meno*,† in which a square is divided into isosceles right triangles. Other critics surmise that the Pythagorean proof was substantially the same as that given by the Hindu astronomer Bhaskara (about 1150 A. D.), who draws the right triangle four times in the square upon its hypotenuse, so that in the middle there remains a square whose side equals the difference between the two sides of the right triangle. Arranging the small square and the four triangles in a different way, they can be shown, together, to make up the sum of the squares of the two sides. In another place Bhaskara gives a second

*M. Cantor, *Vorlesungen ueber Geschichte der Mathematik*, Vol. I, 1894, page 64.
†Cantor, *op. cit.* page 205.

demonstration of this theorem by drawing from the vertex of the right triangle a perpendicular to the hypotenuse and then suitably manipulating the proportions yielded by the similar triangles. This proof was unknown in Europe until it was rediscovered by the English mathematician, John Wallis.

Among Arabic authors the earliest proof, for the case of the isosceles right triangle, was giveu by Alchwarizmî, who lived in the early part of the 9th century. It is the same as that in Plato's *Meno*. The Persian mathematician, Nasîr Eddîn, who flourished during the early part of the 13th century, gave a new proof, which required the consideration of eight special cases.* Until six years ago this proof was attributed to more recent writers.

The theorem of Pythagoras has received several nicknames. In European universities of the Middle Ages it was called "magister matheseos," because examinations for the degree of A. M. (when held at all) appear usually not to have extended beyond this theorem, which, with its converse, is the last in the first book of Euclid. The name, "pons asinorum," has sometimes been applied to it, though usually this is the sobriquet for *Euclid*, I., 5. Some Arabic writers, Behâ Eddîn for instance, call the Pythagorean theorem, "figure of the bride." Curiously enough, this romantic appellation appears to have originated from a mistranslation of the Greek word νύμφη, applied to the theorem by a Byzantine writer of the 13th century. This Greek word admits of two meanings, "bride" and "winged insect." The figure of the right triangle with the three squares suggests an insect, but Behâ Eddîn apparently translated the word as "bride."†

*See H. Suter in *Bibliotheca Mathematica*, 1892, pages 3 and 4.
†See P. Tannery in *L'Intermediare des Mathematiciens*, 1894, Vol. I, page 254.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

106. Proposed by **ELMER SCHUYLER**, High Bridge, N. J.

What is the amount of $1000 at compound interest for three years, at 6%, if it be compounded every instant?

I. Solution by **G. B. M. ZERR**, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa., and **J. OWEN MAHONEY**, B. E.. M. Sc., Instructor in Mathematics, Carthage High School, Carthage, Texas.

Let A=amount, P=principal, r=rate, n=number of years, q=number of times interest is payable a year.

Then $A=P[1+(r/q)]^{qn}$. Let $q=rx$.

$\therefore A=P[1+(1/x)^{nrx}=P\{[1+(1/x)]^x\}^{rn}=Pe^{rn}$ when x is infinite.

$\therefore A=1000e^{\frac{9}{50}}=1000\times1.19705=\$1197.05$.